

BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2008

Senior Final, Part B

Friday, May 2

1. (a) Simplify the expression over a common denominator

$$\frac{1}{a-1} + \frac{1}{a+1}$$

for $a \neq 1$.

- (b) If a is any real number with $a > 1$, use your result in part (a) to determine which is larger

$$\frac{1}{a-1} + \frac{1}{a+1} \quad \text{or} \quad \frac{2}{a}$$

- (c) Use your answer in part (b) to determine which is larger

$$\frac{1}{999999} + \frac{1}{1000001} \quad \text{or} \quad \frac{1}{500000}$$

2. (a) Find the point(s) of intersection of the circle of radius a with centre at the point $(0, a)$ on the y -axis with the parabola $y = x^2$.
- (b) Find the radius of the largest circle with centre on the positive y -axis that intersects the parabola $y = x^2$ only at $(0, 0)$.
3. A census taker, who was very intelligent, came to a house where the three inhabitants were not at home. A neighbour, who knew the inhabitants, was in the yard next door. The census taker already knew the age of the neighbour. To determine the ages, as a whole number of years, of the inhabitants of the house, the census taker asked the neighbour the following questions and received the corresponding replies:

Question: "What is the product of their ages?"

Reply: "252"

Question: "What is the sum of their ages?"

Reply: "The same as their house number."

Question: "Are any of them older than you?"

Reply: "No."

With that information the census taker wrote down the three ages. What are the ages of the inhabitants of the house?

4. The three sides of a triangle have lengths k , $\frac{1}{k}$, and 2. Using a pair of inequalities of the form $a < k < b$, describe the set of all possible values of k .
5. The shortest path on the surface of a cube from vertex A to the furthest vertex B involves crossing a certain number of faces and edges of the cube. See the diagram.

- (a) How many faces and how many edges must be crossed?
- (b) How many such shortest paths are there from vertex A to vertex B ?
- (c) If we call the intersection of a shortest path with an edge a "corner", what is the figure formed by the set of all corners?

