

BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2007

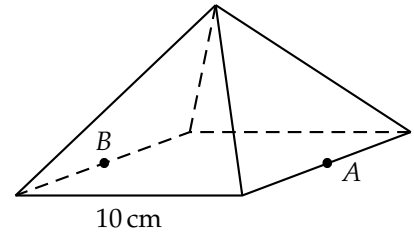
Senior Final, Part B

Thursday May 3

- One of the perpendicular legs of a right triangle has length 11. Given that the lengths of the sides of the triangle are all integers, determine the lengths of the other two sides.
- Consider the first three pairs of sums

$$\begin{aligned} 1 + 2 &= 3 \\ 4 + 5 + 6 &= 7 + 8 \\ 9 + 10 + 11 + 12 &= 13 + 14 + 15 \\ &\vdots \end{aligned}$$

- Determine the largest number on the right hand side of the tenth pair of sums.
 - Determine the value of the sum on each side of the tenth pair of sums.
 - Find a formula for the sum on each side of the k^{th} pair of sums.
- A pyramid made from a solid block of wood rests with its square base on a flat table top. Each edge of the square base has length 10 cm. The lateral sides of the pyramid are equilateral triangles. An ant is walking on the sides of the pyramid from the midpoint A of one of the edges of the base to the midpoint B of the opposite edge. See the diagram. Find the length of the shortest path the ant has to walk.



- Five people are trapped on the island of Survival. Each person can form an alliance with any of the other people on the island. The alliances are mutual, so that if A is allied with B , then B is allied with A . However, if A is allied with B and B is allied with C , it is not necessarily true that A and C are allied. If every person on the island is allied with every other person on the island, then there are ten pairs of people on the island who each have the four alliances.
 - Is it possible that no two people on the island have the same number of alliances? Explain.
 - What is the smallest possible number of pairs of people on the island with the same number of alliances?

- In the diagram, triangle ABC has sides of length $\overline{AB} = 7$, $\overline{AC} = 12$, and $\overline{BC} = 10$. There is a point D on BC such that the circles inscribed in triangles ABD and ACD are both tangent to line AD at a common point E . Find the length of the line segment BD .

