BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2013

Senior Final, Part A

Friday, May 3

1. A train engine has a maximum speed of 120 km/h when no cars are attached. With cars attached, its maximum speed is diminished by a quantity proportional to the square root of the number of cars. With four cars attached its maximum speed is 90 km/h. The largest number of cars that the engine can move is:

   (A) 127 (B) 63 (C) 33 (D) 31 (E) 16

2. The equation $||5x - 4| - 6| = 6$ has n real solutions x. The value of n is:

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. In the diagram start in the bottom left cell and move to the top right cell, adding the numbers as you go. You may move only up and to the right. The largest possible sum is:

   (A) 34 (B) 30 (C) 32 (D) 31 (E) 29

9	2	6	3
1	4	5	7
7	3	2	5
2	8	4	1

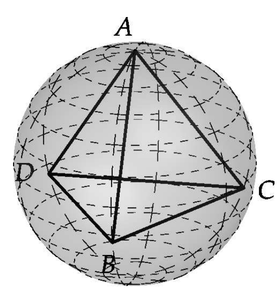
4. Two integers are selected from the set $\{-20, -19, -18, \ldots, 8, 9\}$. If repetitions are allowed, the probability that the product of the two integers is positive is:

   (A) $\frac{9}{100}$ (B) $\frac{3}{10}$ (C) $\frac{4}{9}$ (D) $\frac{481}{900}$ (E) $\frac{2}{3}$

5. Two trains are approaching each other on a long straight section of track. One train is going 5 kilometres per hour and the other is going 3 kilometres per hour. At the time when the trains are 3 kilometres apart a mosquito starts flying from the front of the slower train towards the faster train. When it reaches the faster train it immediately turns around and flies back towards the slower train. If the trains are one kilometre apart when the mosquito first returns to the slower train, the speed at which the mosquito is flying, measured in kilometres per hour, is:

   (A) 8 (B) 12 (C) 15 (D) 16 (E) None of these

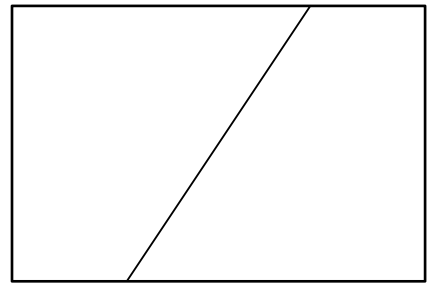
6. A regular tetrahedron has four faces, four vertices and six edges. The edges are of equal length and the faces are all equilateral triangles. A regular tetrahedron inscribed in a sphere has all of its vertices on the surface of the sphere. The volume of the regular tetrahedron inscribed in a sphere of radius one is:



   (A) $\frac{8\sqrt{3}}{27}$ (B) $\frac{8\sqrt{2}}{27}$ (C) $\frac{\sqrt{3}}{4}$

   (D) $\frac{8\sqrt{2}}{12}$ (E) $\frac{\sqrt{3}}{3}$

7. A rectangular piece of paper is folded so that two of its diagonally opposite corners coincide. If the crease is the same length as the longer of the two sides, the ratio of the longer side of the sheet to the shorter side of the sheet is:



(A) $\dfrac{\sqrt{5}-1}{2}$ (B) $\sqrt{\dfrac{\sqrt{5}-1}{2}}$ (C) $\dfrac{\sqrt{5}+1}{2}$

(D) $1+\sqrt{5}$ (E) $\sqrt{\dfrac{\sqrt{5}+1}{2}}$

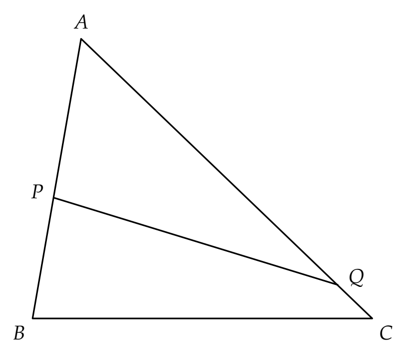
8. The floor in a room can be covered with n square tiles. If a smaller tile is used, then 76 extra tiles are required. If the dimensions of the tiles are integers with no common factors and the number n is an integer, the value of n is:

(A) 324 (B) 342 (C) 360 (D) 400 (E) 684

9. The sum of all of the integer values of z such that $z^2+13z+3$ is n^2, where n is an integer, is:

(A) -46 (B) -13 (C) 33 (D) 39 (E) 157

10. In triangle ABC the sides are $AB=5$, $BC=6$, and $AC=7$. Points P and Q are located on sides AB and AC, respectively, such that $PA+AQ$ equals half the perimeter of triangle ABC, and the area of triangle APQ is half the area of triangle ABC. The length of the line segment PB is:



(A) $\dfrac{9-\sqrt{11}}{2}$ (B) $\dfrac{1+\sqrt{11}}{2}$ (C) 2

(D) 2.5 (E) 3

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Senior Final, Part B

Friday, May 3

1. (a) Write any two digit number and then follow that number by adjoining its reversal. For example, if you start with 13, then you would get 1331. Show that the resulting number is always divisible by 11.

   (b) Show that there are no two digit numbers for which the number is equal to the sum of the squares of its digits.

2. Factor the expression $x^5 - 2x^4 + 3x^3 - 3x^2 + 2x - 1$ completely.

3. Consider the circle $(x-5)^2 + (y-3)^2 = 25$ and the parabola with equation $y = k + a\,(x-h)^2$.

   (a) The circle intersects the x-axis at points A and B. Find their coordinates.

   (b) Determine the value of the constant h so that the parabola intersects the circle at the points A and B.

   (c) Determine the range of possible values for the constant k so that the parabola intersects the circle only at the points A and B.

   (d) Find an expression for the constant a in the equation of the parabola and write the equation of the parabola in terms of x and k alone.

4. The ratio of men and women voters in an election was $a : b$. Had c fewer men and d fewer women voted, the ratio would have been $e : f$. Determine the total number of people who voted as an expression in terms of a, b, c, d, e, and f.

5. In the diagram $ABCD$ is a unit square. The line segment PQ makes an angle θ with side AB and is a diameter of the shaded semicircle. The semicircle is tangent to the side BC of the square of the at point X. Express the area of the semicircle in terms of the angle θ and determine the value of θ for which the area is maximum while still guaranteeing the the semicircle is entirely inside the square.