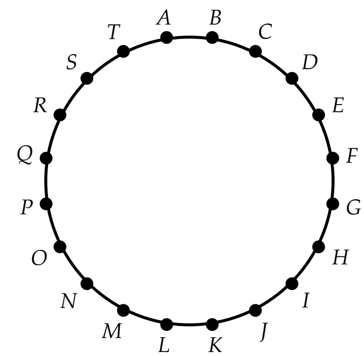
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2012

Senior Final, Part A

Friday, May 4

1. Twenty balloons are equally spaced around a circle. Arnold first pops balloon D and then he pops balloon H. After that he proceed clockwise around the circle popping every fourth unpopped balloon. When there are just three unpopped balloons left, the balloons that are still unpopped are:



(A) P,S,T (B) B,K,R (C) I,Q,R (D) A,G,Q (E) J,P,T

2. A standard *deck* of playing cards consists of 52 *cards* partitioned into 4 *suits* ($\heartsuit$, $\clubsuit$, $\diamondsuit$, and $\spadesuit$) of 13 cards each. The cards in each suit have denominations from (low) 2, 3, . . . , Queen, King, to Ace (high). In the game of bridge the 52 cards are dealt into 4 *hands* of 13 cards each. In order to evaluate the strength of a hand in bridge, the high cards are assigned *high card points* as follows: Jack $= 1$, Queen $= 2$, King $= 3$, and Ace $= 4$. The maximum possible number of points in a randomly dealt 13-card hand is:

(A) 10 (B) 16 (C) 28 (D) 37 (E) 40

3. If $\dfrac{87}{17} = w + \dfrac{1}{y + \dfrac{1}{x}}$, where x, y, and w are all positive integers, then the value of $x + y + w$ is:

(A) 21 (B) 19 (C) 17 (D) 16 (E) 15

4. An integer is prime if it is greater than one and is divisible only by one and itself. By Fermat's Little Theorem, if p is an odd prime number then

$$x = \frac{2^{p-1} - 1}{p}$$

is an integer. The value of x is a perfect square for some values of p. The sum of the values of $p < 15$ for which x is a perfect square is:

(A) 1 (B) 9 (C) 10 (D) 21 (E) 24

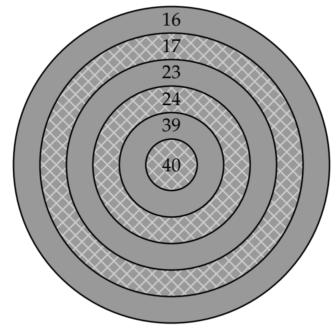
5. The integers from 1 to n are added to form the sum N and the integers from 1 to m are added to form the sum M, where $n > m + 1$. If the difference between the two sums is $N - M = 2012$, then the value of $n + m$ is:

(A) 507 (B) 505 (C) 504 (D) 502 (E) 501

6. The value of $x = 2\sqrt{4 + 2\sqrt{4 + 2\sqrt{4 + 2\sqrt{4 + \cdots}}}}$ is:

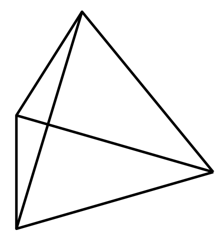
(A) $2\left(1+\sqrt{5}\right)$ (B) $\frac{1}{2}\left(1+\sqrt{17}\right)$ (C) $\frac{1}{2}\left(1+\sqrt{33}\right)$
(D) $2+\sqrt{10}$ (E) $4\left(1+\sqrt{5}\right)$

7. The scores for hitting each ring on a dart board are as shown in the diagram. Assuming that a player can hit any ring she wants on any throw, the smallest number of darts that the player must throw to score exactly 100 is:



(A) 3 (B) 4 (C) 5
(D) 6 (E) 7

8. Six 2 cm pieces of wire are connected together to form a tetrahedron. (See the diagram.) The tetrahedron is placed inside a sphere in such a way the each of its four vertices lie on the sphere. The radius of the sphere is:

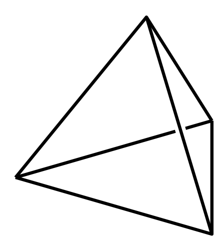


(A) $\dfrac{\sqrt{3}}{3\sqrt{2}}$ (B) $\dfrac{\sqrt{3}}{2\sqrt{2}}$ (C) $\dfrac{\sqrt{3}}{\sqrt{2}}$
(D) $\dfrac{2\sqrt{2}}{\sqrt{3}}$ (E) 2

9. Two railway trains, one 120 metres long and the other 60 metres long, run on parallel tracks, each at a constant speed. When the trains move in opposite directions, the time from the instant when the front ends of the two trains coincide to the instant when their rear ends coincide is 5 seconds. When they move in the same direction, the time from the instant when front end of the faster train coincides with the rear end of the slower train to the instant when the rear end of the faster train coincides with the front end of the slower train is 15 seconds. The speed of the slower train is:

(A) 10 (B) 12 (C) 16 (D) 18 (E) 24

10. Six pieces of wire, all of the same length, are connected together to form a tetrahedron. See the diagram. Each edge of the tetrahedron is to be painted one of two different colours. The number of distinguishable ways in which the tetrahedron can be painted is:



(A) 12 (B) 16 (C) 32
(D) 48 (E) 64

BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2012

Senior Final, Part B

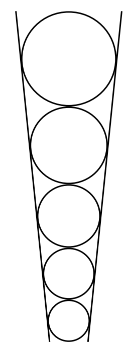
Friday, May 4

1. Let n be any three digit number for which n appears as the last three digits of n^2. Determine all possible values of the number n.

2. Prove that for every positive integer n,

$$\sqrt{n+1} - \sqrt{n} < \frac{1}{2\sqrt{n}} < \sqrt{n} - \sqrt{n-1}$$

3. Five marbles of various sizes are placed in a conical funnel. Each marble is in contact with the adjacent marble(s) and with the funnel wall. The smallest marble has a radius of 8 mm. The largest marble has a radius of 18 mm. Determine the radius, measured in mm, of the middle marble.



4. Mr Baker, Ms Carpenter, Mr Driver, and Ms Plumber are employed as a baker, cook, driver, and plumber. None of them has a name identifying their occupation. They make four statements:

   1. Mr Baker is the plumber.
   2. Mr Driver is the baker.
   3. Ms Carpenter is not the plumber.
   4. Ms Plumber is not the carpenter.

   If all of these statements are true, then Ms Carpenter must be the carpenter, which contradicts the requirement that none of four people have a name that identifies their occupation. Assuming that three of the four statements are false, who is the driver?

5. A square sheet of paper is gray on one side and cross hatched on the other. It lies on a table with the gray side up and is folded so that two adjacent vertices fold over to the perpendicular bisector of the line joining the two original vertices. The fold lines go through the other two vertices. (See the diagram.) After the folding, two gray triangles are visible (ignoring the dashed lines), one large and one small. Determine the ratio of the area of the small triangle to the area of the large triangle.