BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2007

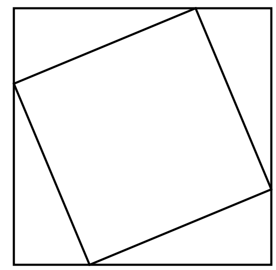
Senior Final, Part A

Thursday May 3

1. The square numbers are: 1, 4, 9, 16, 25, ... The number of square numbers less than 10000 is:

   (A) 99 (B) 100 (C) 101 (D) 499 (E) 500

2. A square of side 13 is inscribed in a square of side 17 as shown in the diagram. The greatest distance between a vertex of the inner square and a vertex of the outer square is:

   (A) 17 (B) $\sqrt{314}$ (C) $\sqrt{433}$

   (D) 22 (E) $\sqrt{458}$

3. A large $n \times n \times n$ cube is formed using n^3 small $1 \times 1 \times 1$ cubes. All six faces of the large cube are painted, the paint is allowed to dry, and the large cube is taken apart into the original n^3 small cubes. The number of small cubes for which at least 2 faces are painted is:

   (A) $8(n-1)$ (B) $4(3n-4)$ (C) $4(3n-2)$ (D) $12n$ (E) $8n$

4. The roots of the equation $x^3 + kx^2 - 1329x = 2007$ are three integers, not necessarily distinct. The value of k is:

   (A) 221 (B) 217 (C) 214 (D) -214 (E) -217

5. A man 2 metres tall is standing at point A on the ground, a certain distance from a lamppost, and observes his shadow. When the man walks 5 metres farther away from the lamppost, his shadow becomes twice as long. The distance along the ground between point A and the lamppost is:

   (A) 5 (B) 10 (C) 12 (D) 15 (E) 20

6. If you write all the integers from 1 to 5555, the number of times you write the digit 9 is:

   (A) 500 (B) 550 (C) 555 (D) 665 (E) 1605

7. A basketball with a radius of 20 centimetres sits in the corner of a room, touching the floor and two walls simultaneously. A softball sits on the floor between the basketball and the corner. The softball touches the floor, the two walls, and the basketball. The radius of the softball, in centimetres, is:

   (A) $20\left(2-\sqrt{3}\right)$ (B) $\dfrac{20}{\sqrt{3}}$ (C) $20\left(2+\sqrt{3}\right)$

   (D) $\dfrac{20}{\sqrt{3}+1}$ (E) $20\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

8. The remote control for Eve's television picks randomly one of the first ten channels 1, 2, 3, ..., 10 in such a way that each channel is equally likely to be picked. Eve's favorite channel is channel 8. She decides to enter it repeatedly using the remote, until she gets channel 8. The probability that she will have to try at most three times is:

(A) $\frac{3}{10}$ (B) $\frac{271}{1000}$ (C) $\frac{1}{10}$ (D) $\frac{81}{1000}$ (E) $\frac{1}{1000}$

9. Suppose that x, y, and z are three distinct positive real numbers and that

$$\frac{y}{x-z} = \frac{x+y}{z} = \frac{x}{y}$$

Then the value of $\frac{x}{y}$ is:

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) $\frac{5}{2}$

10. The distance from point A to the line segment BC is 10 cm. Two lines ℓ and m, parallel to BC, divide the triangle ABC into three parts of equal area. The distance between lines ℓ and m is:

(A) $10\left(1-\frac{\sqrt{3}}{3}\right)$ (B) $10\frac{\sqrt{3}}{3}$ (C) $10\frac{\sqrt{6}}{3}$

(D) $10\frac{\sqrt{3}}{3}\left(\sqrt{2}-1\right)$ (E) $10\left(1-\frac{\sqrt{6}}{3}\right)$