BRITISH COLUMBIA COLLEGES

Junior High School Mathematics Contest, 2004

Final Round, Part B

Friday May 7, 2004

1. The numbers greater than 1 are arranged in an array, in which the columns are numbered 1 to 5 from left to right, as shown:

(1)	(2)	(3)	(4)	(5)
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
⋮	⋮	⋮	⋮	⋮

   (a) In which column will 2004 fall?

   (b) In which column will 1999 fall?

   (c) In which column(s) could $n^2 + 1$ fall, where n is a positive integer?

2. How many sets of two or more consecutive positive integers have a sum of 105?

3. The centres of four circles of radius 12 form a square. Each circle is tangent to the two circles whose centres are the vertices of the square that are adjacent to the centre of the circle. A smaller circle, with centre at the intersection of the diagonals of the square, is tangent to each of the four larger circles. Find the radius of the smaller circle.
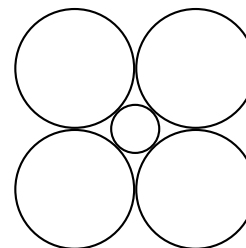
4. The Fibonacci sequence begins: $1, 1, 2, 3, 5, 8, 13, 21, \ldots$. (Each number beyond the second number is the sum of the previous two numbers.) The notation f_n means the nth number; for example $f_4 = 3$ and $f_7 = 13$.

   (a) Which of the following terms in the Fibonacci sequence are odd? Explain your conclusions.

   $$f_{38}, \ f_{51}, \ f_{150}, \ f_{200}, \ f_{300}$$

   (b) Which of the following terms in the Fibonacci sequence are divisible by 3? Explain your conclusions.

   $$f_{48}, \ f_{75}, \ f_{196}, \ f_{379}, \ f_{1000}$$

5. The diagram shows three squares. Find the measure of the angle $\alpha + \beta$.