BRITISH COLUMBIA COLLEGES

Junior High School Mathematics Contest, 2004

Final Round, Part A

Friday May 7, 2004

1. The sum of the squares of the lengths of the three sides of a right triangle is 10. The length of the hypotenuse is:

   (a) $\sqrt{5}$ (b) 3   (c) 5   (d) 2   (e) none of these

2. You travel from Kamloops to Vancouver at $80\,\text{km/h}$ and return along the same route at $100\,\text{km/h}$. Your average speed, to the nearest km/h, for the complete journey is:

   (a) 88   (b) 91   (c) 90   (d) 92   (e) 89

3. A number is called an *increasing number* if each digit in the number is greater than the digit to its left. The number of increasing numbers between 5000 and 10000 is:

   (a) less than 4   (b) 4   (c) 5   (d) 6   (e) more than 6

4. Given that, for a natural number n, $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$, the units digit of $S = 1! + 2! + 3! + \cdots + 50!$ is:

   (a) 0   (b) 1   (c) 3   (d) 8   (e) 9

5. In the diagram each box must contain exactly one of the symbols shown and each symbol must occur once and only once. The clues below determine which symbol appears in which box.

       1 is to the right of 4;   A is not below ♡;   D is beside C;
       ♡ and ○ are not adjacent;   ☺ is not beside ♭;
       4 and 1 are separated by one box;
       + and C are in the same column.

   The third column is:

6. The value of $\dfrac{2^{2004} - 2^{2003}}{2^{2004} + 2^{2003}}$ is:

   (a) 0   (b) 2^{-2006} (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{2}$ (e) 2^{2002}

7. A circle which has its centre in the first quadrant is tangent to the y-axis and intersects the x-axis at $(3,0)$ and $(9,0)$. The area of that portion of the circle which is not is the first quadrant is:

   (a) $6\pi - 9\sqrt{3}$ (b) $12\pi - 9\sqrt{3}$ (c) $\dfrac{3\pi}{2} - \dfrac{9\sqrt{3}}{4}$ (d) $3\pi - \dfrac{9\sqrt{3}}{4}$ (e) none of these

8. Suppose (a, b) is a solution to the system of equations $ab = 5$ and $a^2b + ab^2 + a + b = 42$. The value of $a^2 + b^2$ is:

    (a) 6   (b) 7   (c) 36   (d) 39   (e) 49

9. The triangle ABC is an isosceles right triangle with hypotenuse BC of length $4\sqrt{2}$ cm. A 10 cm string is attached to the triangle at point C as in the diagram. The string is kept straight while it is wrapped around triangle ABC in a <u>clockwise</u> direction. The area, in square centimetres, swept out by the string is:

    (a) 57π (b) 54π (c) 50π

    (d) 48π (e) 44π
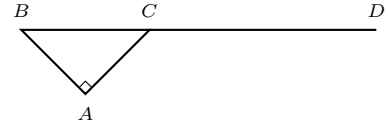
10. The area of square $ABCD$ is 2. The diagonal AC is extended to triple its own length to point E, where C is between A and E. The distance BE is:

    (a) $\sqrt{10}$ (b) 4   (c) $\sqrt{26}$ (d) $4 + \sqrt{2}$ (e) $5\sqrt{2}$