BRITISH COLUMBIA COLLEGES

Junior High School Mathematics Contest, 2005

Final Round, Part A

Friday May 6, 2005

1. Two operations $\star$ and $\diamond$ are defined by the two tables below:

$\star$	1	2	3
1	1	3	2
2	1	3	1
3	3	3	1

$\diamond$	1	2	3
1	4	2	3
2	3	6	5
3	2	6	4

For example, $1 \diamond 2 = 2$. The value of $2 \diamond (3 \star 3)$ is:

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

2. Three people leave their coats in a check room. When they check out, three coats are distributed randomly among them. The probability that *none* of the three receives the correct coat is:

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$

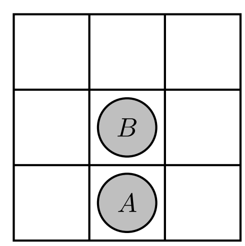
3. A grocer uses a pan balance on which weights can be placed on either of the pans together with the object being weighed. The grocer has three weights that will balance precisely any whole number of kilograms from 1 kg to 13 kg. The three weights are:

(A) 2, 5, 6 (B) 3, 4, 6 (C) 1, 5, 7 (D) 2, 4, 7 (E) 1, 3, 9

4. The number of cards that must be drawn from a standard deck of 52 playing cards to be sure that at least two are aces or three are of the same suit is:

(A) 9 (B) 13 (C) 27 (D) 49 (E) 50

5. The game of Solitaire JumpIt is played on a 3×3 grid. A single player places two or more game discs on the grid. If two discs, A and B, are adjacent horizontally, vertically, or diagonally and there is an open space on the side of B away from A, then A can jump B and disc B is removed. See the diagram. The player makes jumps as long as possible. The player wins if he or she can continue until only one disc remains. The maximum number of discs that can be placed on the grid in a way that the player still wins is:



(A) 3 (B) 4 (C) 5
(D) 6 (E) 7

6. The product $\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1-\frac{1}{n-1}\right)\left(1+\frac{1}{n}\right)$ is equal to:

(A) 1 (B) $\frac{1}{n}$ (C) $\frac{1+n}{n}$ (D) -1 (E) None of these

7. The number of integers between 500 and 600 which have 12 as the sum of their digits is:

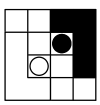
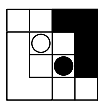
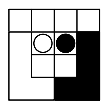
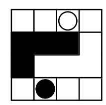
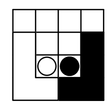
   (A) 6 (B) 7 (C) 8 (D) 10 (E) 12

8. A grid point in the plane is a point (x, y) for which both x and y are integers. The number of grid points that lie within or on the boundary of the region bounded by the parabola $y = x^2$ and the line $y = 50$ is:

   (A) 470 (B) 485 (C) 490 (D) 750 (E) 765

9. *Wot th'ell* is a game played on a 4×4 checker board. Both players have an *L*-shaped piece which covers four squares and a disc which covers one. The players alternate moves, one playing white pieces and the other playing black. A move consists of picking up the *L*-shaped piece, possibly turning it over, and placing it back on the board in a new position. Then the player removes his disk and puts it back on the board (the disk may be returned to where it came from.) Neither the *L*-shaped piece nor the disc can be placed so that it covers any square that is already occupied by a disc or an *L*-shaped piece. A player who is unable to move loses.

   The one of the following boards on which white can play and win is:

   (A)  (B)  (C)  (D)  (E) 

10. There is a critical height (which is a whole number of floors above ground level), such that an egg dropped from that height (or higher) will break, but if dropped from a lower height (no matter how many times), it will not break. You are given two eggs and told that the critical height is between 1 floor and 37 floors (inclusive). You want to develop a plan that gives the most efficient way to determine this critical height. Obviously, starting at the first floor and going up one floor at a time could require only one drop, if the critical height is one; but it could require as many as 37 drops, if the critical height is 37. The optimum plan will give the smallest value for the maximum possible number of drops. The maximum possible number of drops required by the optimum plan for determining the critical height is:

    (A) 8 (B) 9 (C) 12 (D) 19 (E) 35