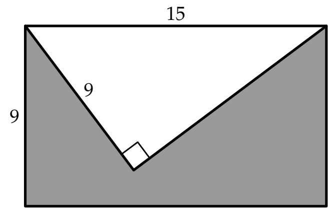
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2014

Junior Final, Part A

Friday, May 2

1. The rectangle shown has sides of lengths 9 and 15. The length of one side of the internal right triangle is 9. The area of the shaded region is:



(A) 54 (B) 81 (C) 108
(D) 120 (E) 135

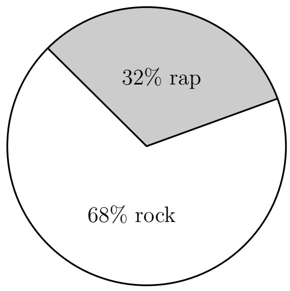
2. In the subtraction below, a and b are distinct digits with $a < b$.

$$\begin{array}{r} 2\,0\,1\,4 \\ -\quad a\,5 \\ \hline 1\,9\,b\,9 \end{array}$$

If neither a nor b equals any of the other digits, then the value of digit b is:

(A) 3 (B) 6 (C) 7 (D) 8 (E) Cannot be determined

3. Adrian's music playlist contains 68% rock and 32% rap, as shown in the pie chart. Adrian wants to add some jazz so that 25% of the playlist is jazz, while the ratio of rock to rap stays the same. After the jazz is added, the percentage of the playlist that is rap is:



(A) 20 (B) 24 (C) 25
(D) 30 (E) 32

4. A total of 480 students take part in an on-line game tournament. Each student participates in exactly 4 different games. Each of the games in the tournament has exactly 20 student participants and is supervised by 1 adult coach. There are 16 adult coaches and each coach supervises the same number of games. The number of games supervised by each coach is:

(A) 16 (B) 15 (C) 12 (D) 8 (E) 6

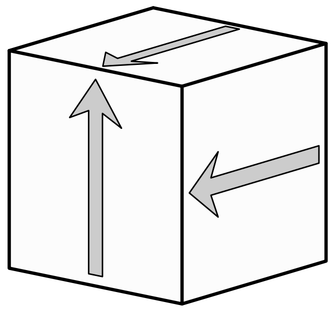
5. The number 123 456 789 is multiplied by the number 999 999 999. The number of times the digit 8 appears in the result is:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6. An integer $n \geq 2$ is said to be **practical** if every $m = 1, 2, 3, \ldots, n-1$ can be written as a sum of some distinct divisors of n. Otherwise, n is **impractical**. For example, the divisors of 4 are 1 and 2, and $1 = 1$, $2 = 2$, and $3 = 1 + 2$, so that 4 is practical. The smallest *even* impractical number is:

(A) 18 (B) 12 (C) 10
(D) 6 (E) No even integer is impractical.

7. The cube shown has a picture of an arrow on each of its faces, each one pointing to the centre of one edge of the face. Notice that the arrows on the front and top faces meet tip-to-tip. If the three unseen arrows are oriented randomly in any of the four possible directions, the probability that no other arrows on the cube meet tip-to-tip is:



(A) $\frac{13}{16}$ (B) $\frac{3}{4}$ (C) $\frac{11}{16}$

(D) $\frac{5}{8}$ (E) $\frac{9}{16}$

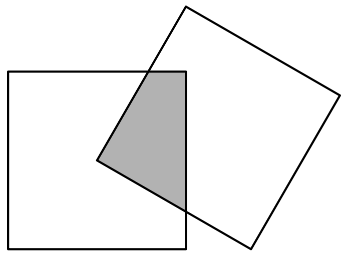
8. A wire is cut into two pieces of equal length. One piece is bent to form an equilateral triangle with an area of 2, and the other is bent to form a regular hexagon. The area of the hexagon is:

(A) 2 (B) $\frac{3}{2}\sqrt{3}$ (C) 3 (D) $2\sqrt{3}$ (E) 4

9. Ronnie drives for exactly 5 hours to complete a trip to visit his grandparents. During the trip Ronnie drives along a highway passing through five towns. It takes exactly 5 minutes to drive through each town at an average speed of 60 kilometres per hour. The overall average speed for the entire trip is 100 kilometres per hour. Ronnie's average highway speed, measured in kilometres per hour, when not driving through a town is:

(A) $103\,\frac{7}{11}$ (B) 105 (C) 112 (D) 140 (E) $103\,\frac{1}{2}$

10. Two squares are situated so that the corner of one square is at the centre of the other (see the diagram). The fraction of the area of the leftmost square that is shaded is:



(A) $\frac{3}{16}$ (B) $\frac{1}{5}$ (C) $\frac{5}{24}$

(D) $\frac{1}{4}$ (E) Cannot be determined

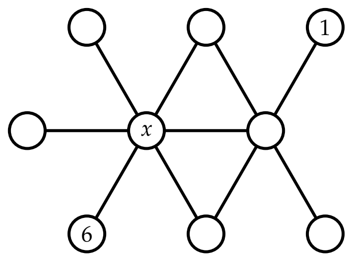
BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2014

Junior Final, Part B

Friday, May 2

1. Find an expression for each of the numbers 2 through 10 using all of the digits from 2014 exactly once, in the order they appear in 2014, together with any of the mathematical operations of addition $(+)$, subtraction $(-)$, multiplication $(\times)$, division $(\div)$, negation (multiplication by -1), exponentiation, or square root $(\sqrt{\ })$ and using brackets ( ), as required. Note that concatenation of the digits is not allowed, for example, $20 - 14 = 6$ or $\sqrt{2 + 0 + 14} = 4$ are not acceptable, since they involve the concatenation of the digits 2 and 0 to make 20 or 1 and 4 to make 14.

2. In the diagram, each of the integers 1 through 9 is to be placed in one circle so that the integers in every straight row of three joined circles, both horizontal and diagonal, add to 18. The 6 and 1 have been filled in as shown. Determine the value of the number represented by x in the diagram. Justify your answer.



3. (a) Consider three positive integers x, y and z each greater than 1. If $xyz = 49,000$ and each pair of integers, namely, (x, y), (x, z) and (y, z), have a greatest common factor of 1, determine the value of $x + y + z$.

   (b) Prove that the square of any odd integer always leaves a remainder of 1 when divided by 8.

4. The *sumset* $S + S$ is the set of all distinct sums of any pair of numbers, possibly the same number, taken from a set S.

   (a) Show that the sumset of the set $S = \{3, 5, 6\}$ contains six distinct numbers.

   (b) Define the set $S = \{a, b, c\}$ where a, b, and c are positive integers with $a < b < c$. Show that there are at least five distinct numbers in the sumset $S + S$.

   (c) If two of the three numbers in the set S are 15 and 21, find all possible values of the third number for which $S + S$ contains exactly five distinct numbers.

5. Two identical large coins and two identical small coins are positioned as shown. (Where they appear to be tangent, they are.) The minimum (vertical) distance between the small coins is 2 mm, and the minimum (horizontal) distance between the large coins is 6 mm. If the radius of the small coin is r and the radius of the large coin is R, express r in terms of R. Express your answer in the form $r = \dfrac{aR + b}{cR + d}$ where a, b, c, and d are integers.