BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2012

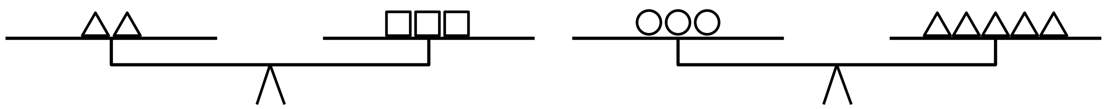
Junior Final, Part A

Friday, May 4

1. When an integer n is divided by 7 the remainder is 6. The remainder when $6n$ is divided by 7 is:

   (A) 6 (B) 5 (C) 3 (D) 2 (E) 1

2. The diagram shows two equal arm balance beams.



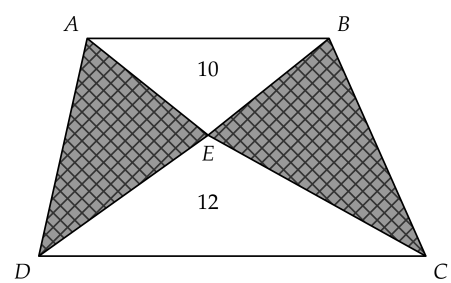
   The number of □'s required to balance ○○ is:

   (A) 3 (B) 5 (C) 6 (D) 8 (E) 10

3. A community group has 500 members. At their spring dance, new members paid only $14 for a ticket, but longtime members paid $20 per ticket. Consequently all the new members attended but only 70% of the longtime members attended. The total revenue collected from ticket sales was:

   (A) $7000 (B) $10000 (C) $12000 (D) $14000 (E) Impossible to determine.

4. The point E lies in the interior of the trapezoid $ABCD$. The areas of triangles ABE and CDE are 10 and 12, respectively, and the length of line segment AB is two-thirds of the length of line segment CD. The total area of the shaded region (triangle ADE plus triangle BCE) is:



   (A) 20 (B) 23 (C) 24

   (D) 45 (E) 54

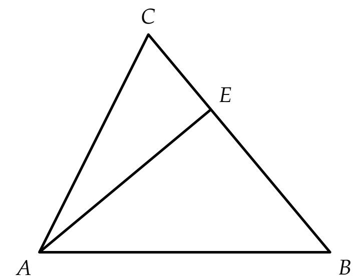
5. The integers from 1 to n are added to form the sum N and the integers from 1 to m are added to form the sum M, where $n > m + 1$. If the difference between the two sums is $N - M = 2012$, then the value of $n + m$ is:

   (A) 507 (B) 505 (C) 504 (D) 502 (E) 501

6. The four digits 0, 1, 2, and 2 can be arranged to form twelve different four digit numbers. Note that some of the numbers will have zero as the leading digit. If the resulting twelve numbers are listed from the least to the greatest, the position of the number 2012 is:

   (A) 5$^{\text{th}}$ (B) 7$^{\text{th}}$ (C) 8$^{\text{th}}$ (D) 10$^{\text{th}}$ (E) 11$^{\text{th}}$

7. In triangle ABC line segment AE is an altitude perpendicular to side BC. Further, $|\overline{CE}| = 2$ and $|\overline{EB}| = 6$. If the area of triangle ABC is 20, then $|\overline{AB}|$ equals:



(A) 7 (B) $\sqrt{52}$ (C) $\sqrt{61}$

(D) 8 (E) None of these

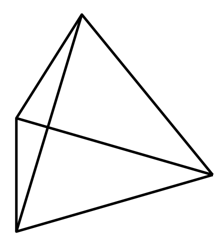
8. The number of times between noon and midnight when the hour hand and the minute hand of a clock are at right angles to each other is:

(A) 10 (B) 12 (C) 21 (D) 22 (E) 24

9. Marni is selling six gift cards online. She has only one of each card and wants to sell as many as she can. The dollar values of the cards are \$30, \$32, \$36, \$38, \$40, and \$62. The first buyer purchases two cards and the second buyer spends twice as much money as the first buyer. The amount of money Marni received in total from the two buyers is:

(A) \$198 (B) \$204 (C) \$186 (D) \$304 (E) \$222

10. Six 2 cm pieces of wire are connected together to form a tetrahedron. (See the diagram.) The shortest distance from one vertex of the tetrahedron to the opposite face is:



(A) $\dfrac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) $\dfrac{2\sqrt{2}}{\sqrt{3}}$

(D) $\dfrac{\sqrt{2}}{\sqrt{3}}$ (E) 2

BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2012

Junior Final, Part B

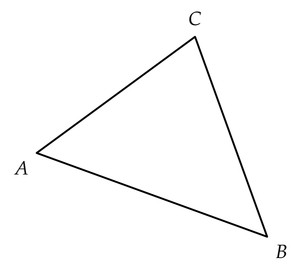
Friday, May 4

1. In the sum at the right each different letter represents a distinct digit and none of the numbers in the sum has a zero as the leading digit. Determine the digit represented by the letter T.

$$\begin{array}{rr} & \text{F O R T Y} \\ & \text{T E N} \\ + & \text{T E N} \\ \hline & \text{S I X T Y} \end{array}$$

2. Define the operation $\star$ as $a \star b = \dfrac{a+2b}{2}$. Simplify the expression $(a \star b) \star c - a \star (b \star c)$.

3. Triangle ABC has sides with integer length and its area is an integer. One side of the triangle has length 21, and the perimeter of the triangle is 48. Find the length of the shortest side.



4. Prove that $\dfrac{1}{\sqrt{2}-1} < 2\sqrt{2} < \dfrac{1}{\sqrt{3}-\sqrt{2}}$.

5. A dyslexic bank teller transposed the number of dollars and cents when he cashed a cheque for Ms Smith, giving her dollars instead of cents and cents instead of dollars. After buying a newspaper for 50 cents, Ms Smith noticed that she still had exactly three times as much as the original cheque. Determine the original amount of the cheque.