

**BRITISH COLUMBIA SECONDARY SCHOOL
MATHEMATICS CONTEST, 2011**

Junior Final, Part A

Friday, May 6

1. The digits 1, 2, 3, 4 can be arranged to form twenty-four different four-digit numbers. If these twenty-four numbers are then listed from the smallest to largest, the number in the 14th position is:

(A) 2431 (B) 3124 (C) 3142 (D) 3241 (E) 4123

2. Some three digit numbers satisfy the following conditions:

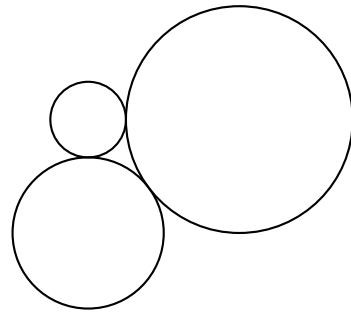
- The sum of the digits is 13.
- The product of the digits is greater than 60.

The number of such three digit numbers is:

(A) 4 (B) 8 (C) 12 (D) 15 (E) 18

3. Three circular coasters rest on a tabletop, each coaster just touching the other two. (See the diagram.) The radii of the coasters are, respectively, 1, 2, and 3 centimetres. The area, measured in square centimetres, of the triangle whose vertices are the centres of the coasters is:

(A) $2\sqrt{10}$ (B) $4\sqrt{2}$ (C) 6
(D) $4\sqrt{3}$ (E) 8



4. Three women go to a restaurant for lunch, and each leaves her hat on a hat stand. When they leave the restaurant, each woman picks up a hat at random. The probability that each woman picks up her own hat is:

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{8}$ (E) $\frac{1}{5}$

5. Consider the sequence a_1, a_2, \dots, a_n for which $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{a_n}$, for $n \geq 1$. The smallest value of n (greater than 2) for which the **denominator** of a_n , after a_n is written in lowest terms, is a perfect square, is:

(A) 9 (B) 12 (C) 17 (D) 29 (E) No such n exists

6. The number of points of intersection of the circle $x^2 + y^2 = 9$ and parabola $y = x^2 - 3$ is:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. A man was traveling through a desert. At noon he arrived at a nomadic village where he was received by two hospitable hosts A and B, who brought some pita bread from their tents to share with their guest. Host A brought three pita bread and host B brought five pita bread. The three men shared the bread equally and after the meal the traveler continued his journey, but before leaving he handed eight gold coins to A and asked him to share it fairly with B. Since B had brought five pita bread A handed B five gold coins and kept three for himself. However, B demanded more gold coins as he felt that the distribution of the coins was not fair. The number of additional gold coins B should have received was:

- (A) -1 (B) 0 (C) 1 (D) 2 (E) 3

8. Three distinct positive digits A , B , and C satisfy the equation $A^B + B^A = 10A + C$. The sum of the digits $A + B + C$ equals:

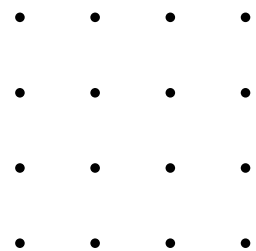
- (A) 10 (B) 12 (C) 13 (D) 14 (E) 17

9. A soup can rests on a flat horizontal surface. The can is a right circular cylinder with a circumference of 12 cm and a height of 5 cm. A bug located at a point on the circumference of the base wishes to crawl to the point on the circumference of the top of the can directly above it by spiraling up the surface of the can, making one complete circuit around the can. The shortest distance, measured in centimetres, that the bug can go to complete its trip is:

- (A) $\frac{12}{\pi} + 5$ (B) 13 (C) 5π (D) 17 (E) $\sqrt{221}$

10. Consider the array of dots shown. The total number of squares whose vertices are selected from the dots shown is:

- (A) 9 (B) 13 (C) 17
 (D) 19 (E) 20

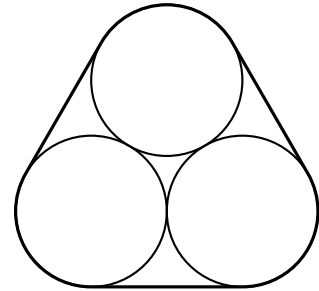


BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2011

Junior Final, Part B

Friday, May 6

1. Alexa has an unopened bag of jellybeans. After opening the bag she gives her brother half the jellybeans, and then eats 13 of the remaining jellybeans. Her sister then eats one more than half of the remaining jellybeans, after which there are 3 left. Determine the number of jellybeans in the bag before it was opened.
2. Three circular coins, all with radius 1 cm, are mutually tangent. An elastic band is stretched snugly around them. (See the diagram.) Determine the length, measured in centimetres, of the stretched elastic.



3. Find all solutions to the equation

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$

for which x and y are positive integers.

4. Four married couples visit Pepper Island. The husbands are named Archie, Bruce, Chuck, and Dave; the wives are Amanda, Bernadette, Caroline, and Diane. The inhabitants of Pepper Island use the google as the unit of currency and one google is worth 21 yahoos. Each of the eight visitors goes to a different sheep ranch and buys a certain number of sheep. The amount that each person pays per sheep, in yahoos, exactly equals the number of sheep he or she buys. When they meet at the ferry terminal to go home, they find that for each couple the husband spent exactly five googles more than his wife. Further, Archie bought 11 more sheep than Amanda; Caroline bought 5 more sheep than Chuck; and Dave bought 9 more sheep than Bernadette. To whom is Amanda married?
5. A square sheet of paper $ABCD$ of area 12 cm^2 is blue on one side and red on the other. It is placed on a table with the blue side down and corner A is folded over to a point A' so that a blue isosceles triangle is now visible. If the area of the blue triangle equals the area of the remaining red region, determine the distance between the point A' and the vertex C .

