BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2010

Junior Final, Part A

Friday, May 7

1. An operation consists of doubling a number and then subtracting 1. This operation is carried out 30 times starting with the number 3. The final value is:

   (A) $2^{31}+1$ (B) $3 \cdot 2^{30}+1$ (C) $3 \cdot 2^{30}$ (D) $3 \cdot 2^{31}$ (E) $2^{31}-1$

2. The number of pairs (m, n), where m and n are positive integers, that satisfy the equation $m\,(m+1) = 2^n$ is:

   (A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

3. A company wants to construct a rectangular box that will hold exactly 150 cubes each of dimension $1 \times 1 \times 1$ centimetre. The minimum possible surface area of the box, measured in square centimetres, is:

   (A) 120 (B) 160 (C) 170 (D) 190 (E) 230

4. At this time, the largest known prime number is $2^{43112609}-1$. It requires 13 million digits to write in decimal form. If the length of 4 printed digits is 1 centimetre, the number of kilometres required to print the largest prime is closest to:

   (A) 5 (B) 9 (C) 11 (D) 21 (E) 32

5. Observe that $800 = 30^2 - 10^2$, and, hence, that 800 can be expressed as the difference of the squares of two integers. Of the following numbers the number that cannot be expressed as the difference of the squares of two integers is:

   (A) 40 (B) 41 (C) 42 (D) 43 (E) 44

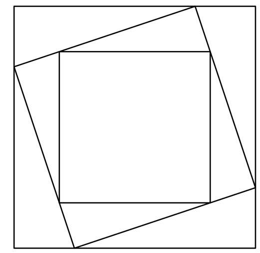
6. The number of ways to make change for one dollar using only nickels, dimes, and quarters, using at least one of each is:

   (A) 9 (B) 10 (C) 11 (D) 13 (E) 15

7. The awards officer at a certain college receives scholarship applications from four students. Each student is awarded a scholarship in a different amount. Letters are sent out to each student announcing his or her award, but the letters are accidentally mixed up so that each student receives the letter for a different student. The number of ways in which this can happen is:

   (A) 15 (B) 12 (C) 9 (D) 6 (E) 1

8. The diagram shows three nested squares. The middle square is positioned so that each of its corners divides a side of the outer square into segments whose length are in ratio 3 : 1. Similarly, the inner square is positioned so that each of its corners divides the side of the middle square into segments whose lengths are in ratio 3 : 1. The ratio of area of the inner square to the area of the outer square is:



(A) 9 : 16 (B) 25 : 81 (C) 4 : 9

(D) 25 : 64 (E) 1 : 2

9. Suppose that

$$\frac{97}{19} = w + \cfrac{1}{x + \cfrac{1}{y}}$$

where w, x, and y are positive integers. The value of $w + x + y$ is:

(A) 14 (B) 16 (C) 19 (D) 21 (E) 26

10. In a certain school 25% of the students are blue-eyed and 75% are brown-eyed. Also, 10% of the blue-eyed students are left-handed, and 5% of the brown-eyed students are left-handed. The percentage of left-handed students who are blue-eyed is:

(A) 10 (B) 15 (C) 20 (D) 30 (E) 40

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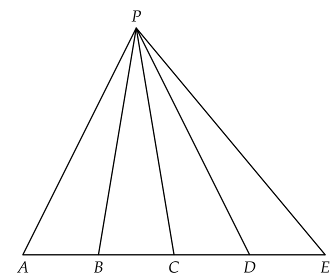
Junior Final, Part B

Friday, May 7

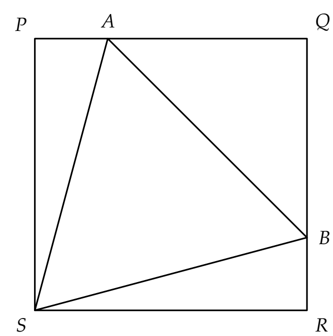
1. (a) Find the sum of all positive whole numbers less than 2010 for which the units digit is either a '3' or an '8'.

   (b) Two cans X and Y both contain some water. From X Tim pours as much water into Y as Y already contains. Then, from Y he pours as much water into X as X already contains. Finally, he pours from X into Y as much water as Y already contains. Each can now contains 24 units of water. Determine the number of units of water in each can at the start.

2. The area of triangle APE shown in the diagram is 12. Given that $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$, determine the sum of the areas of all the triangles that appear in the diagram.



3. Given that $PQRS$ is a square and ABS is an equilateral triangle (See the figure.), find the ratio of the area of triangle APS to the area of triangle ABQ.



4. Find the five distinct integers for which the sums of each distinct pair of integers are the numbers 0, 1, 2, 4, 7, 8, 9, 10, 11, and 12.

5. A rectangle contains three circles, as in the diagram, all tangent to the rectangle and to each other. The height of the rectangle is 4. Determine the width of the rectangle.